Options for path following method

Generalities

In the path following method, the external loading is parameterized by a scalar, so-called load factor λ, which is considered as an additional unknown of the problem. The balance equation reads

‘$\mathbf{F}_{int}(\mathbf{U}, \mathbf{Z}) = \lambda \mathbf{F}_{ext},$‘

where ‘$\mathbf{U}$‘ is the unknown vector and ‘$\mathbf{Z}$‘ is the vector including all historical variables to follow path-dependent processes. To close the problem statement, an additional equation is added:

‘$\Phi(\mathbf{U}, \lambda, ...) = 0.$‘

In the code, there exists two different possibilities:

- Hyper-spherical control: the addition equation is built directly based on the degree of freedom under a hyper-spherical form:

‘$\Phi = a_{\mathbf{U}} \Delta\mathbf{U}^T \Delta\mathbf{U} + a_\lambda \Delta\lambda^2 - \Delta s^2.$‘

where Δs is the arc-length increment, and where $a_{\mathbf{U}}$ and a_λ are two constants. There also exists different possibilities:

- Arc-length control: this name is abused for three different cases:
  - ‘$a_{\mathbf{U}} = 1$‘ and ‘$a_\lambda = 0$‘: state control
  - ‘$a_{\mathbf{U}} = 1$‘ and ‘$a_\lambda = 1$‘: arc-length control
  - ‘$a_{\mathbf{U}} = 0$‘ and ‘$a_\lambda = 1$‘: load control (this case corresponds to the usual load increment Newton-Raphson procedure).
- Hyperelliptic control: in general, each DOF in ‘$\mathbf{U}$‘ has different nature (displacement, nonlocal, etc.). In this case, we can apply the path following control for several components (defined by the component of a DOF in the solver, e.g. 0,1,2 corresponding to displacement field, etc)

‘$\Phi = \sum_i a_{\mathbf{U}}^i \Delta\mathbf{U}_i^T \Delta\mathbf{U}_i + a_\lambda \Delta\lambda^2 - \Delta s^2.$‘

The value of ‘$a_{\mathbf{U}}^i$‘ can be specified to obtain a desired control.

- Irreversible energy-based control: the addition equation is built based on the dissipation energy. It is advantageous for problems involving dissipation. This method must be combine with a load control at the begining at which there is no dissipation.

- Automatic time stepping: the path following should be used with the automatic time stepping. The automatic time step is estimated by

Some useful options

Arc-length control

Irreversible energy-based control

Hyperelliptic control

More details about path following options

To activate the path following method

By supposing the solver `mysolver` has been created by

```
#------------------------------
mysolver=nonLinearMechSolver(tag)
# tag can be an arbitrary integer
```

The path following can be activated using

```
#------------------------------
mysolver.pathFollowing(const bool p, const int method)
# p must be True to activate the path following
# method must be chosen between 0 and 1
#        =0 if a global arc-length control is used
#        =1 if a dissipation-based control is used
#        =2 if a hyper elliptic control is used
```

Once a method is actived, the options for each method need to be specified as follows.

Global arc-length control

In case of the global arc-length control, the following options can be considered:

- The kind of constraints

```
#------------------------------
mysolver.setPathFollowingControlType(const int type)
# type (1 by default) must be speficified between 0, 1, or 2 as
#   = 0 for a pure state control
#   = 1 for an arc-length control
#   = 2 for a pure load control
```

- When considering `method=0 or 1`, we can specify the correction method by

```
#------------------------------
mysolver.setPathFollowingCorrectionMethod(const int corrmethod)
# corrmethod (1 by default) must be speficified between 0, 1, 2
#            = 0 if the main system (i.e. Fint = Fext)
#                and path following constrol are solved simultanously
#            = 1 if the path following is solved after the main system
#            = 2 if the path following is solved after the main system
#                and a safe condition for the control equation is considered.
```